Abstract

Introduction to 3 dimensional space

1 12.1 Three Dimensions

Three dimensional coordinate system uses three real lines (but half lines are easier to draw) called axes passing through the origin and perpendicular to each other. The axes are labelled x, y and z or one, two and three and must be oriented using the "right hand rule." Axis 1 (x-axis) is positive in the direction of the right index finger; axis 2 (y-axis) is in the direction of the right index finger and axis 3 (the z-axis) points in the direction of the right thumb. The choice of axes involves choosing an origin and the first axis and a perpendicular second axis and then the third axis is determined. Any point in 3 space is determined assuming a scale has been established on each axis (which may differ from axis to axis).

Example: Plot (2,-1,-3) on a set of coordinate axes.

Therefore three dimensional space is identified with ordered triples (x, y, z) of real numbers ($x, y, z \in \mathbb{R}$ and we write $\mathbb{R}^3$ for the set of all such triples. Similarly we define $\mathbb{R}^2$.

Example: What is the distance between (2,1) and (3,5)? What is the distance between (2,1,-1) and (3,5,2)? Diagram:

Distance Formula: The distance between $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$|PQ| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Therefore the distance between $P(-2, 3, 6)$ and $Q(2, -1.8)$ is 6.

Example: The equation

$$(x - 2)^2 + (y - 5)^2 + (z + 1)^2 = 9$$

is satisfied if and only if $P(x, y, z)$ is a distance 3 from (2,5,-1) and so it is the equation of a sphere of radius 3 centered at (2,5,-1). Usually sphere refers to the surface of a ball and ball refers to the solid: $(x-2)^2+(y-5)^2+(z+1)^2 \leq 9$.

Example: Find the set of all points equidistant from (1,3,5) and (2,-1,0).